

# Lecture 2. for Week 9, March 23 – 29

*This lecture presumes that you have read chapter 9 in your text. Remember, when we say "read" ... we mean read-read. Reading Physics is going to ruin your literature reading speed. Sorry!* 

Today's lecture focuses on the concepts and tasks that tend to trip students up.

### 1<sup>st</sup>: The Dot Product

• We have said that once you go 2 or 3 dimensional in Geometry ... you really have added only one new idea ... and that's the abstract notion of <u>direction</u>. You already are intimately familiar with the idea of comparing magnitudes (i.e. numbers) ... because you do that on even a single number line and have done so since grade school. The really new thing, then, is comparing <u>directions</u>. Having the same direction is called "parallel" and having <u>completely</u> NOT the same direction is called "parallel" and having <u>completely</u> NOT the same direction is called "parallel" and having <u>completely</u> NOT the same direction is called "perpendicular". In some deep sense ... this is all that is going on in Geometry. If we choose to assemble our ideas about Geometry into the structure we are now calling Vector Algebra, then determining just how much two vectors, let's call them  $\vec{A} \otimes \vec{B}$ , are "in the same direction or not" ... is the most important thing there is to say. Actually, it's just about the only thing there is to say. This is why the dot and cross products are so important! They tell us exactly this information. The dot product gives the product of parallel parts and the cross product gives the product of perpendicular parts. The Physical World is built squarely on these ideas. Don't imagine that you can somehow weasel your way out and evade these ideas ... the truth is ... you have to hug them! Work is a <u>dot-product</u> ... a product of the *Displacement along a Force* ... i.e. a product of the parallel parts of these two foundational entities :  $dW \equiv \vec{F} \cdot d\vec{r}$ .

What so confuses many a young student is that we have multiple representations for these things... three in fact. We have a Component/Cartesian expression, we have a Polar/Euclidean expression and also, finally, a wholly Abstract/Gibbsian representation which we write  $\overrightarrow{A} \cdot \overrightarrow{B}$ .

 $\overrightarrow{A} \cdot \overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$  yes, but <u>also</u> ...  $\overrightarrow{A} \cdot \overrightarrow{B} = AB \cos(\theta_{included})$ 

Where A and B are the respective magnitudes of the vectors  $\overrightarrow{A}$  and  $\overrightarrow{B}$ . That these two expressions give exactly the same output is really the same truth as the cosine double angle formula. I show this in substantial depth (i.e....more than most of you wanted ... though you won't find expositions such as these elsewhere on the web and I hope you come to appreciate or ...dare I hope ... even "cherish" some distant day) in the notes **Pre-Introduction to Vector Geometry** found on our web-site: https://physics.csuchico.edu/buchholtz/4A/204A vectorIntro.pdf

Three expressions ... but one meaning: *"Projection"* of one vector on another. Learning to manipulate these abstract vector symbols will make you mighty! It will solve all your problems like nothing else ... and is one of the rock-bottom skills of our trade.

#### 2nd: Work

The central technical features of the concept *Work* are illustrated in the following simple picture:



The key points to grasp:

- Each participating force may be doing work individually.
- *N* and *mg* are perpendicular to the displacement and will do no work even though they are very much present and do affect the motion. They have no projection along the motion.
- Only the <u>component</u> of the applied force *F* along the displacement will do work (the projection along...) ... in this case the applied force *F* has a <u>positive</u> component along the direction of displacement and does <u>positive</u> work.
- The kinetic frictional force  $f_k$  does work ... but since it points *opposite* to the displacement ... its work is <u>negative</u> in sign.
- With the definition of work comes a new dimensionality:  $(mass)(length/time)^2$  and a new S.I. unit ... the Joule  $\Leftrightarrow 1J = 1kg^1 m^2 s^{-2}$ . The Joule is a fairly "practical" unit in that daily occurrences don't require massive prefixes ... except for nutrition!

## **3rd:** Power

The concept power exists only to help us keep track of energy. Power is defined as work done per time. In symbols:  $P \equiv \Delta W / \Delta t$ . A universal expression then for the instantaneous power i.e. the energy delivered per time is  $dW/dt \equiv \vec{F} \cdot d\vec{r}/dt$  or  $P \equiv \vec{F} \cdot \vec{v}$  which we will use frequently.

Points to keep in mind are:

• Power only has interest inasmuch as energy has interest for the problem.

• Power is not a conserved quantity! You don't "buy" power ...although many people speak of it that way. You buy energy.

• Power has a new dimensionality: (mass)(length)<sup>2</sup>(time)<sup>-3</sup> and a new S.I. unit ... the Watt.

 $1W = 1 \text{kg} (\text{meter})^2 (\text{second})^{-3}$  An ordinary lightbulb might be 60W (that usage doesn't bother you), but in the future you're going to have to get used to everything – including cars – being rated in S.I. units. Consider that a modern Honda Accord's engine is rated at about 188kW. That probably bothers you ... and thinking of a modern passenger car as 3,000 lightbulbs is a bit weird. What costs more 3,000 LED lightbulbs ... or a passenger car? Which lasts longer? "Investment cost" per Watt delivered?

#### 4th: Applications

A key idea that is coming to the fore here is watching which forces do work. Suppose that a box slides down a frictionless hillside as shown:



The Normal force is very much present and keeps the block from "sinking into" the hill ...yes, ... but it is everywhere perpendicular to the velocity ... and thus perpendicular to the momentary displacement at each moment! It does no work.

Does the hill have to be smooth ...? How about:



No change in the argument! In the descent, the normal force does no work ... and thus transfers no energy.

So in a mysterious way, the "Ballistics scenario" from the first week of class where there is no "hill" ... really isn't different from an energy perspective from the frictionless "hills" above. The parabolic ballistics trajectory is seen as "just another frictionless hill".



These are powerful insights into what is really happening in the physical world. Knowledge about mechanical energy isn't all the knowledge there is ... but it's utterly key to solving problems.